Artificial Intelligence

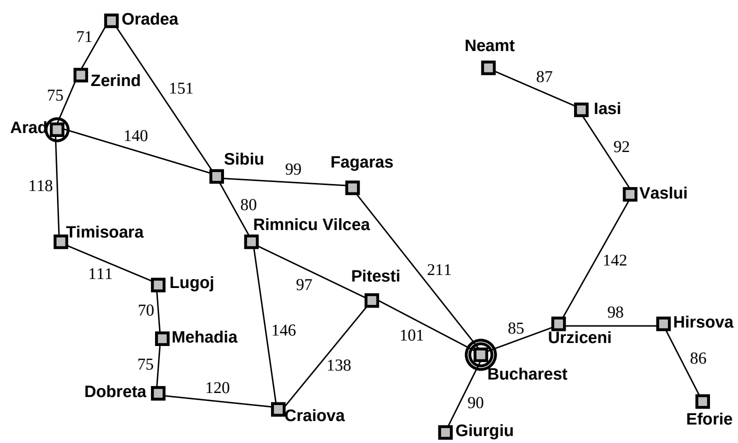
Problem Solving and Search

Readings: Chapter 3 of Russell & Norvig.

Example: Romania

- **Problem**: On holiday in Romania; currently in Arad. Flight leaves tomorrow from Bucharest. Find a short route to drive to Bucharest.
- **Formulate problem**:
  - *states*: various cities
  - *actions*: drive between cities
  - *solution*: sequence of cities, e.g., Arad, Sibiu, Fagaras, Bucharest

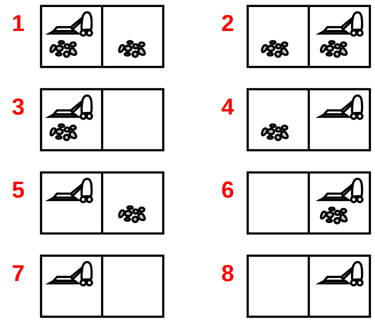
Example: Romania



Problem types

- Deterministic, fully observable $\Longrightarrow$ *single-state problem*
  - Agent knows exactly which state it will be in; solution is a sequence
- Non-observable $\Longrightarrow$ *conformant problem*
  - Agent may have no idea where it is; solution (if any) is a sequence
- Nondeterministic and/or partially observable $\Longrightarrow$ *contingency problem*
  - percepts provide *new* information about current state
  - solution is a *tree* or *policy*
  - often *interleave* search, execution
- Unknown state space $\Longrightarrow$ *exploration problem* ("online")

Example: Vacuum World



Single-state,
start in #5.
Solution??

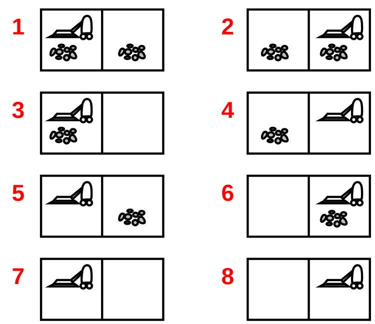
.

Problem Solving

We will start by considering the simpler cases in which the following holds.

- The agent's world (environment) is representable by a **discrete set of states.**
- The agent's actions are representable by a **discrete set of operators.**
- The world is **static** and **deterministic.**

Example: Vacuum World

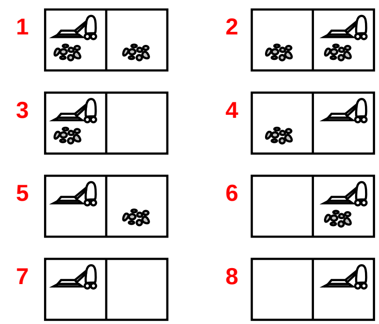


Single-state, start in #5.
Solution?? $[Right, Suck]$

Conformant, start in $\{1, 2, 3, 4, 5, 6, 7, 8\}$
e.g., $Right$ goes to $\{2, 4, 6, 8\}$.
Solution??
$[Right, Suck, Left, Suck]$

Contingency, start in #5

Example: Vacuum World



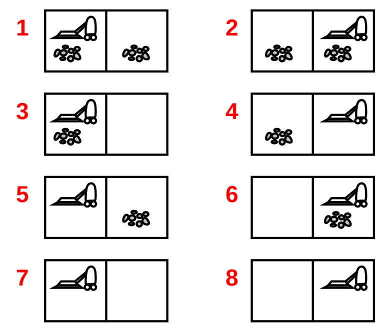
Single-state, start in #5.
Solution?? $[Right, Suck]$

Conformant, start in $\{1, 2, 3, 4, 5, 6, 7, 8\}$
e.g., $Right$ goes to $\{2, 4, 6, 8\}$.
Solution??

Single-state problem formulation

- A *problem* is defined by four items:
  - *initial state*   e.g., "at Arad"
  - *successor function* $S(x)$ = set of action–state pairs e.g., $S(Arad) = \{\langle Arad \rightarrow Zerind, Zerind\rangle, \ldots\}$
  - *goal test*, can be *explicit*, e.g., x = "at Bucharest" *implicit*, e.g., $NoDirt(x)$
  - *path cost* (additive) e.g., sum of distances, number of actions executed, etc. Usually given as $c(x, a, y)$, the step cost from x to y by action a, assumed to be ≥ 0.
- A *solution* is a sequence of actions leading from the initial state to a goal state

Example: Vacuum World

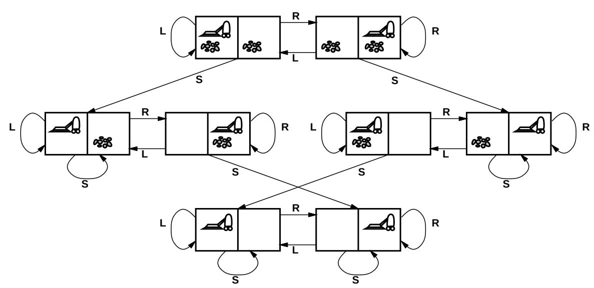


Single-state, start in #5.
Solution?? $[Right, Suck]$

Conformant, start in $\{1, 2, 3, 4, 5, 6, 7, 8\}$
e.g., $Right$ goes to $\{2, 4, 6, 8\}$.
Solution??
$[Right, Suck, Left, Suck]$

Contingency, start in #5

State space graph of vacuum world

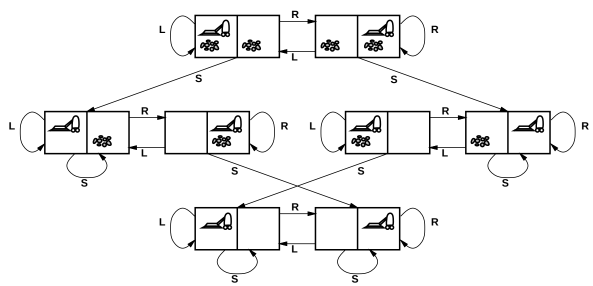


states??
actions??
goal test??
path cost??

Selecting a State Space

- Real world is absurdly complex $\Rightarrow$ state space must be *abstracted* for problem solving
- (Abstract) state = set of real states
- (Abstract) action = complex combination of real actions e.g., "Arad $\rightarrow$ Zerind" represents a complex set of possible routes, detours, rest stops, etc.
  - For guaranteed realizability, any real state "in Arad" must get to *some* real state "in Zerind".
  - Each abstract action should be "easier" than the original problem!
- (Abstract) solution = set of real paths that are solutions in the real world

State space graph of vacuum world



<u>states??</u>: integer dirt and robot locations (ignore dirt *amounts*)
<u>actions??</u>: $Left$, $Right$, $Suck$, $NoOp$
<u>goal test??</u>: no dirt

<u>path cost??</u>: 1 per action (0 for $NoOp$)

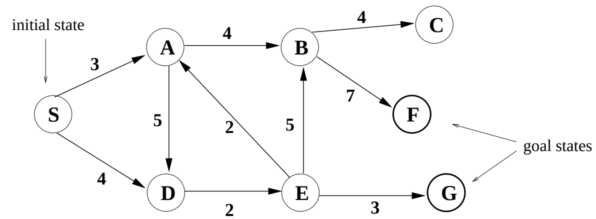
Formulating Problem as a Graph

In the graph

- each node represents a possible **state**;
- a node is designated as the **initial state**;
- one or more nodes represent **goal states**, states in which the agent's goal is considered accomplished.
- each edge represents a **state transition** caused by a specific agent action;
- associated to each edge is the **cost** of performing that transition.

Search Graph

How do we reach a goal state?



There may be several possible ways. Or none!

Factors to consider:

- cost of *finding* a path;
- cost of *traversing* a path.

Problem Solving as Search

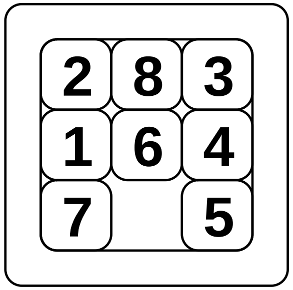
Search space: set of states reachable from an initial state S_0 via a (possibly empty/finite/infinite) sequence of state transitions.

To achieve the problem's goal

- **search** the space for a (possibly optimal) sequence of transitions starting from S_0 and leading to a goal state;
- **execute** (in order) the actions associated to each transition in the identified sequence.

Depending on the features of the agent's world the two steps above can be interleaved.

Example: The 8-puzzle



It can be generalized to 15-puzzle, 24-puzzle, or $(n^2 - 1)$-puzzle for $n \geq 6$.

Problem Solving as Search

- Reduce the original problem to a search problem.
- A solution for the *search problem* is a path initial state–goal state.
- The solution for the *original problem* is either
  - the sequence of actions associated with the path or
  - the description of the goal state.

Example: The 8-puzzle

States: configurations of tiles

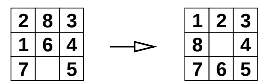
Operators: move one tile Up/Down/Left/Right

- There are $9! = 362,880$ possible states (all permutations of $\{\Box, 1, 2, 3, 4, 5, 6, 7, 8\}$).
- There are $16!$ possible states for 15-puzzle.
- Not all states are directly reachable from a given state. (In fact, exactly half of them are reachable from a given state.)

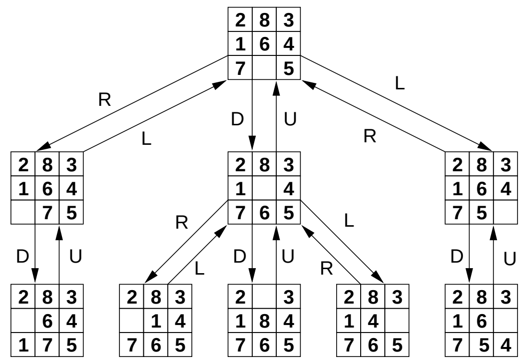
How can an artificial agent represent the states and the state space for this problem?

Example: The 8-puzzle

Go from state S to state G.

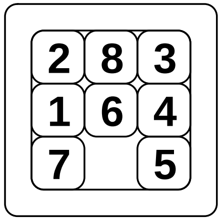


(S) (G)



Formulating the 8-puzzle Problem

States: each represented by a 3×3 array of numbers in $[0 \ldots 8]$, where value 0 is for the empty cell.



becomes A = $\begin{matrix} 2 & 8 & 3 \\ 1 & 6 & 4 \\ 7 & 0 & 5 \end{matrix}$

Problem Formulation

1. Choose an appropriate data structure to represent the world states.
2. Define each operator as a precondition/effects pair where the
   - **precondition** holds exactly in the states the operator applies to,
   - **effects** describe how a state changes into a successor state by the application of the operator.
3. Specify an initial state.
4. Provide a description of the goal (used to check if a reached state is a goal state).

Preconditions and Effects

Example: $Op_{(3,2,R)}$

$$\begin{matrix} 2 & 8 & 3 \\ 1 & 6 & 4 \\ 7 & 0 & 5 \end{matrix} \quad \overset{Op_{(3,2,R)}}{\Longrightarrow} \quad \begin{matrix} 2 & 8 & 3 \\ 1 & 6 & 4 \\ 7 & 5 & 0 \end{matrix}$$

Preconditions: $A[3,2] = 0$

Effects: $\begin{cases} A[3,2] & \leftarrow & A[3,3] \\ A[3,3] & \leftarrow & 0 \end{cases}$

We have 24 operators in this problem formulation . . .

20 too many!

Formulating the 8-puzzle Problem

- **Operators:** 24 operators of the form $Op_{(r,c,d)}$ where $r, c \in \{1,2,3\}$, $d \in \{L, R, U, D\}$.
- $Op_{(r,c,d)}$ moves the empty space at position (r,c) in the direction d.

$$\begin{matrix} 2 & 8 & 3 \\ 1 & 6 & 4 \\ 7 & 0 & 5 \end{matrix} \quad \overset{Op_{(3,2,L)}}{\Longrightarrow} \quad \begin{matrix} 2 & 8 & 3 \\ 1 & 6 & 4 \\ 0 & 7 & 5 \end{matrix}$$

A Better Formulation

Operators: 4 operators of the form Op_d where $d \in \{L, R, U, D\}$.

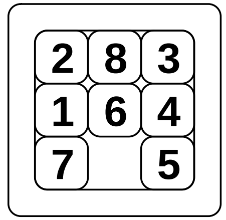
Op_d moves the *empty space* in the direction d.

$$\begin{matrix} 2 & 8 & 3 \\ 1 & 6 & 4 \\ 7 & 0 & 5 \end{matrix} \quad \overset{Op_L}{\Longrightarrow} \quad \begin{matrix} 2 & 8 & 3 \\ 1 & 6 & 4 \\ 0 & 7 & 5 \end{matrix}$$

A Better Formulation

States: each represented by a pair $(A, (i, j))$ where:

- A is a 3×3 array of numbers in $[0 \ldots 8]$
- (i, j) is the position of the empty space (0) in the array.



becomes $\left(\begin{matrix} 2 & 8 & 3 \\ 1 & 6 & 4 \\ 7 & 0 & 5 \end{matrix}, (3, 2) \right)$

Half states are not reachable?

Can this be done?

1	2	3
4	5	6
7	8	

$\overset{any\ steps}{\Longrightarrow}$

1	2	3
4	5	6
8	7	

$1,000 award for anyone who can do it!

Preconditions and Effects

Example: Op_L

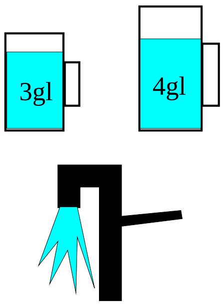
$$\left(\begin{matrix} 2 & 8 & 3 \\ 1 & 6 & 4 \\ 7 & 0 & 5 \end{matrix}, (3, 2) \right) \overset{Op_L}{\Longrightarrow} \left(\begin{matrix} 2 & 8 & 3 \\ 1 & 6 & 4 \\ 0 & 7 & 5 \end{matrix} (3, 1) \right)$$

Let (r_0, c_0) be the position of 0 in A.

Preconditions: $c_0 > 1$

Effects:
$$\begin{cases} A[r_0, c_0] & \leftarrow & A[r_0, c_0 - 1] \\ A[r_0, c_0 - 1] & \leftarrow & 0 \\ (r_0, c_0) & \leftarrow & (r_0, c_0 - 1) \end{cases}$$

The Water Jugs Problem



Get exactly 2 gallons of water into the 4gl jug.

Half states are not reachable?

a_1	a_2	a_3
a_4	a_5	a_6
a_7	a_8	a_9

Let the 8-puzzle be represented by $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9)$. We say (a_i, a_j) is an inversion if neither a_i nor a_j is blank, $i < j$ and $a_i > a_j$.

1	2	3
4	5	6
7	8	

1	2	3
4	5	6
8	7	

The first one has 0 inversions and the second has 1.

Claim: # of inversions modulo two remains the same after each move.

The Water Jugs Problem: Operators

F4: fill jug4 from the pump.

precond: $J_4 < 4$ **effect:** $J'_4 = 4$

E4: empty jug4 on the ground.

precond: $J_4 > 0$ **effect:** $J'_4 = 0$

E4-3: pour water from jug4 into jug3 until jug3 is full.

precond: $J_3 < 3,\ J_4 \geq 3 - J_3$ **effect:** $J'_3 = 3,\ J'_4 = J_4 - (3 - J_3)$

P3-4: pour water from jug3 into jug4 until jug4 is full.

precond: $J_4 < 4,\ J_3 \geq 4 - J_4$ **effect:** $J'_4 = 4,\ J'_3 = J_3 - (4 - J_4)$

E3-4: pour water from jug3 into jug4 until jug3 is empty.

precond: $J_3 + J_4 < 4,\ J_3 > 0$ **effect:** $J'_4 = J_3 + J_4,\ J'_3 = 0$

...

The Water Jugs Problem

States: Determined by the amount of water in each jug.

State Representation: Two real-valued variables, J_3, J_4, indicating the amount of water in the two jugs, with the constraints:

$$0 \leq J_3 \leq 3, \quad 0 \leq J_4 \leq 4$$

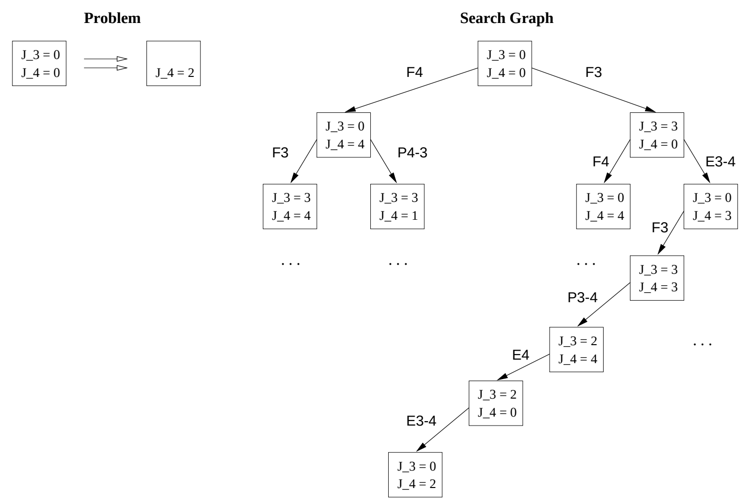
Initial State Description

$$J_3 = 0, \quad J_4 = 0$$

Goal State Description:

$J_4 = 2 \quad \Leftarrow$ non exhaustive description

The Water Jugs Problem



Real-World Search Problems

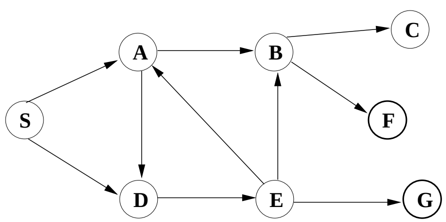
- Route Finding
  (computer networks, airline travel planning system, . . .)
- Travelling Salesman Optimization Problem
  (package delivery, automatic drills, . . .)
- Layout Problems
  (VLSI layout, furniture layout, packaging, . . .)
- Assembly Sequencing
  (assembly of electric motors, . . .)
- Task Scheduling
  (manufacturing, timetables, . . .)

Problem Solution

- Problems whose solution is a *description of how to reach a goal* state from the initial state:
  - n-puzzle
  - route-finding problem
  - assembly sequencing
- Problems whose solution is simply a *description of the goal* state itself:
  - 8-queen problem
  - scheduling problems
  - layout problems

More on Graphs

A graph is a set of notes and edges (arcs) between them.

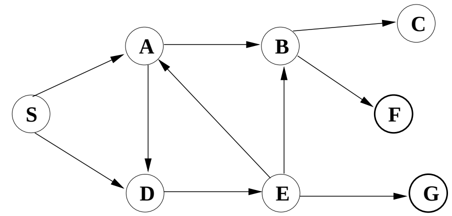


A graph is *directed* if an edge can be traversed only in a specified direction.

When an edge is directed from n_i to n_j

- it is univocally identified by the pair (n_i, n_j)
- n_i is a *parent* (or *predecessor*) of n_j
- n_j is a *child* (or *successor*) of n_i

Directed Graphs



A *path*, of length $k \geq 0$, is a sequence $\langle (n_1, n_2), (n_2, n_3), \ldots, (n_k, n_{k+1}) \rangle$ of k *successive* edges. [a]

Ex: $\langle \rangle, \langle (S, D) \rangle, \langle (S, D), (D, E), (E, B) \rangle$

For $1 \leq i < j \leq k+1$,

- N_i is a *ancestor* of N_j; N_j is a *descendant* of N_i.

A graph is *cyclic* if it has a path starting from and ending into the same node. *Ex:* $\langle (A, D), (D, E), (E, A) \rangle$

[a] Note that a path of length $k > 0$ contains $k+1$ nodes.

From Search Graphs to Search Trees

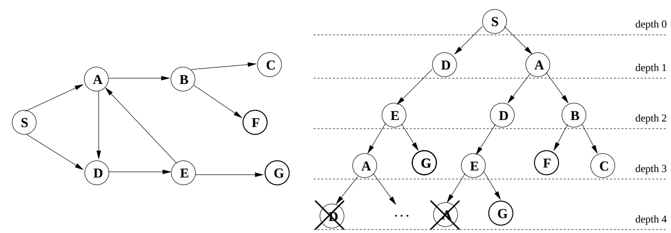
The set of all possible paths of a graph can be represented as a tree.

- A *tree* is a directed acyclic graph all of whose nodes have at most one parent.
- A *root* of a tree is a node with no parents.
- A *leaf* is a node with no children.
- The *branching factor* of a node is the number of its children.

Graphs can be turned into trees by duplicating nodes and breaking cyclic paths, if any.

From Graphs to Trees

To unravel a graph into a tree choose a root node and trace every path from that node until you reach a leaf node or a node already in that path.



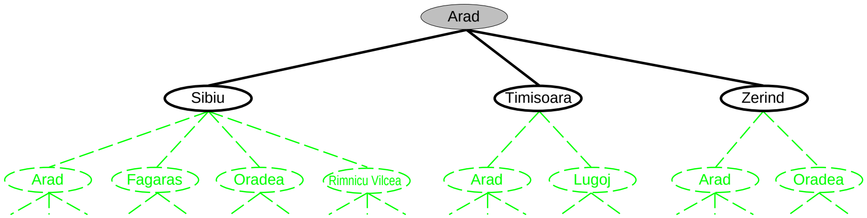
- must remember which nodes have been visited
- a node may get duplicated several times in the tree
- the tree has infinite paths only if the graph has infinite non-cyclic paths.

Tree Search Algorithms

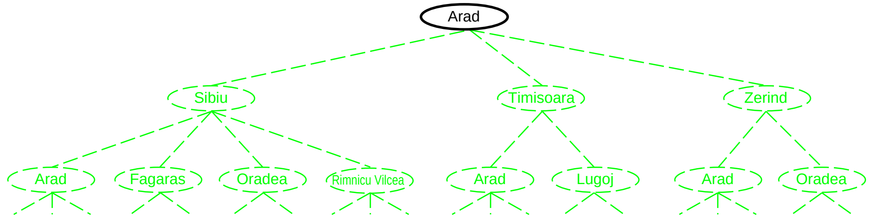
Basic idea: offline, simulated exploration of state space by generating successors of already-explored states (a.k.a. *expanding* states)

```
function TREE-SEARCH( problem, strategy) returns a solution, or failure
   initialize the search tree using the initial state of problem
   loop do
       if there are no candidates for expansion then return failure
       choose a leaf node for expansion according to strategy
       if the node contains a goal state then return the solution
       else expand the node and add the resulting nodes to the search tree
   end
```

Tree search example

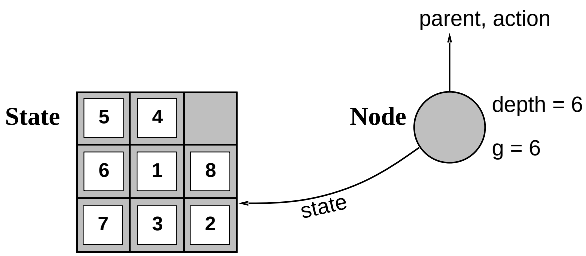


Tree Search Example



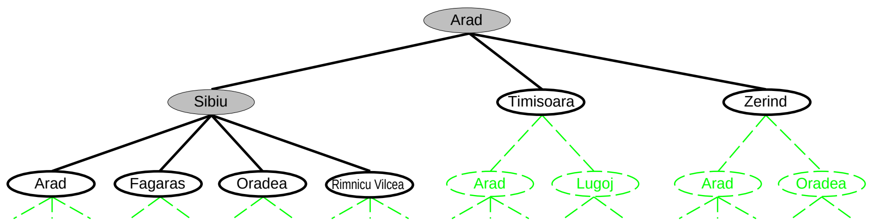
Implementation: states vs. nodes

A *state* is a (representation of) a physical configuration
A *node* is a data structure constituting part of a search tree includes *parent*, *children*, *depth*, *path cost* $g(x)$
States do not have parents, children, depth, or path cost!



The EXPAND function creates new nodes, filling in the various fields and using the SUCCESSORFN of the problem to create the corresponding states.

Tree Search Example



Search Strategies

- A strategy is defined by picking the *order of node expansion*. Strategies are evaluated along the following dimensions:
  - completeness—does it always find a solution if one exists?
  - time complexity—number of nodes generated/expanded
  - space complexity—maximum number of nodes in memory
  - optimality—does it always find a least-cost solution?
- Time and space complexity are measured in terms of
  - b—maximum branching factor of the search tree
  - d—depth of the least-cost solution
  - m—maximum depth of the state space (may be ∞)

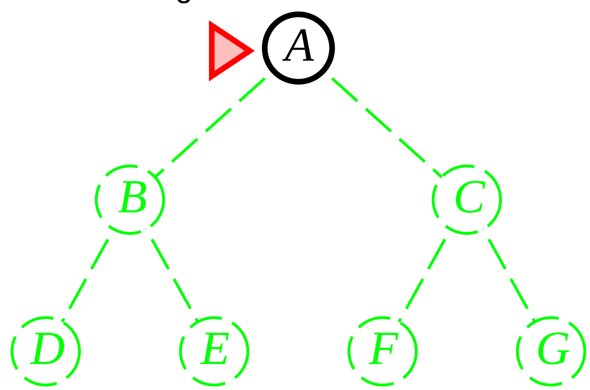
Uninformed Search Strategies

Uninformed strategies use only the information available in the problem definition

- Breadth-first search
- Uniform-cost search
- Depth-first search
- Depth-limited search
- Iterative deepening search

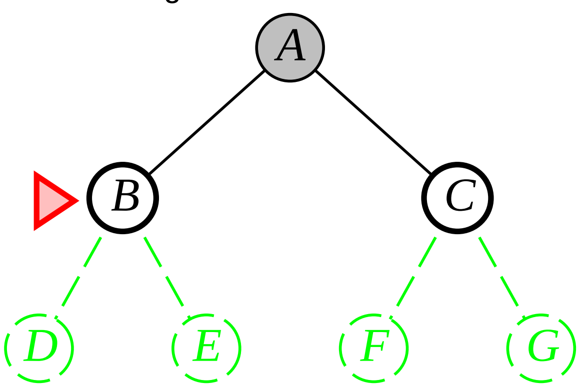
Breadth-First Search

Expand shallowest unexpanded node
Implementation: *fringe* is a FIFO queue, i.e., new successors go at end



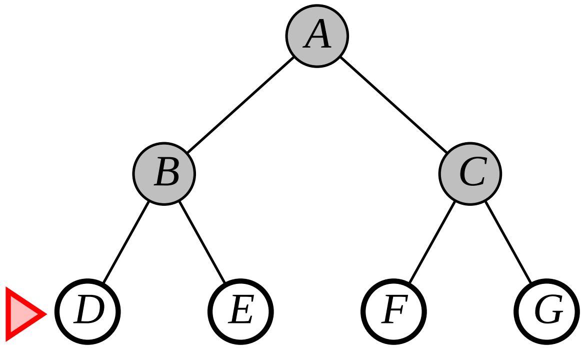
Breadth-First Search

Expand shallowest unexpanded node
Implementation: *fringe* is a FIFO queue, i.e., new successors go at end



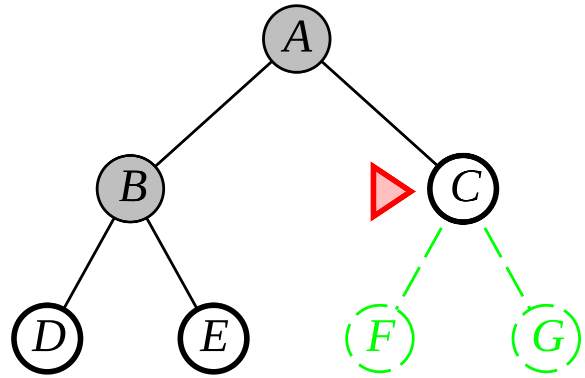
Breadth-First Search

Expand shallowest unexpanded node
Implementation: *fringe* is a FIFO queue, i.e., new successors go at end



Breadth-First Search

Expand shallowest unexpanded node
Implementation: *fringe* is a FIFO queue, i.e., new successors go at end



Properties of Breadth-First Search

Complete?? Yes (if b is finite)
Time??

Properties of Breadth-First Search

Complete??

Properties of Breadth-First Search

Complete?? Yes (if b is finite)

Time?? $1 + b + b^2 + b^3 + \ldots + b^d + b(b^d - 1) = O(b^{d+1})$, i.e., exp. in d

Space?? $O(b^{d+1})$ (keeps every node in memory)

Optimal??

Properties of Breadth-First Search

Complete?? Yes (if b is finite)

Time?? $1 + b + b^2 + b^3 + \ldots + b^d + b(b^d - 1) = O(b^{d+1})$, i.e., exp. in d

Space??

Properties of Breadth-First Search

Complete?? Yes (if b is finite)

Time?? $1 + b + b^2 + b^3 + \ldots + b^d + b(b^d - 1) = O(b^{d+1})$, i.e., exp. in d

Space?? $O(b^{d+1})$ (keeps every node in memory)

Optimal?? Yes (if cost = 1 per step); not optimal in general

Space?? It is the big problem; can easily generate nodes at 10MB/sec so 24hrs = 860GB.

Properties of Breadth-First Search

Complete?? Yes (if b is finite)

Time?? $1 + b + b^2 + b^3 + \ldots + b^d + b(b^d - 1) = O(b^{d+1})$, i.e., exp. in d

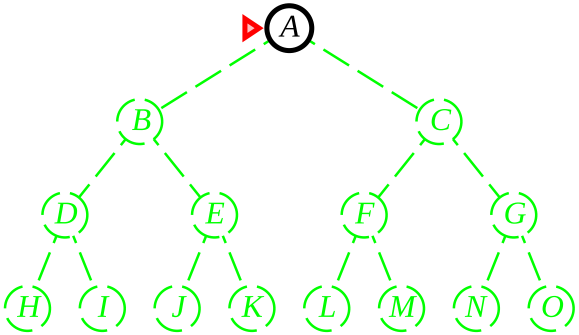
Space?? $O(b^{d+1})$ (keeps every node in memory)

Optimal?? Yes (if cost = 1 per step); not optimal in general

Space??

Depth-First Search

Expand deepest unexpanded node
Implementation: *fringe* = LIFO queue, i.e., put successors at front

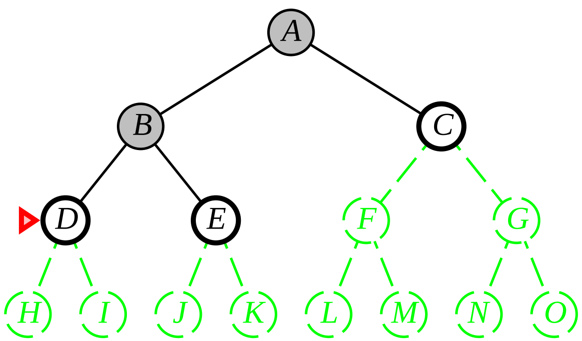


Uniform-Cost Search

Expand least-cost unexpanded node
Implementation: *fringe* = queue ordered by path cost
Equivalent to breadth-first if step costs all equal
Complete?? Yes, if step cost $\geq \epsilon$
Time?? # of nodes with $g \leq$ cost of optimal solution, $O(b^{\lceil C^*/\epsilon \rceil})$ where C^* is the cost of the optimal solution
Space?? # of nodes with $g \leq$ cost of optimal solution, $O(b^{\lceil C^*/\epsilon \rceil})$
Optimal?? Yes—nodes expanded in increasing order of $g(n)$

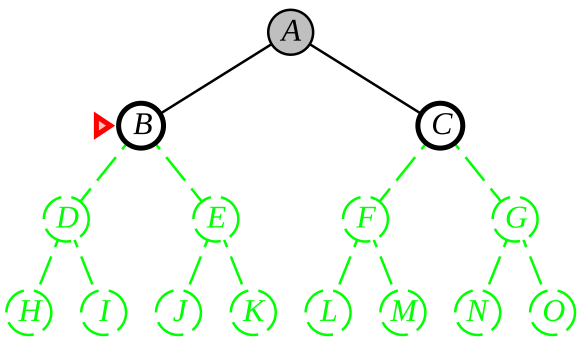
Depth-First Search

Expand deepest unexpanded node
Implementation: *fringe* = LIFO queue, i.e., put successors at front



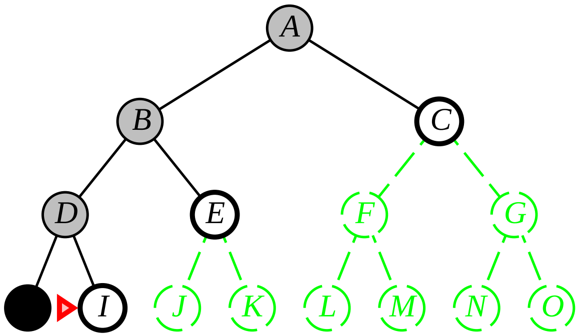
Depth-First Search

Expand deepest unexpanded node
Implementation: *fringe* = LIFO queue, i.e., put successors at front



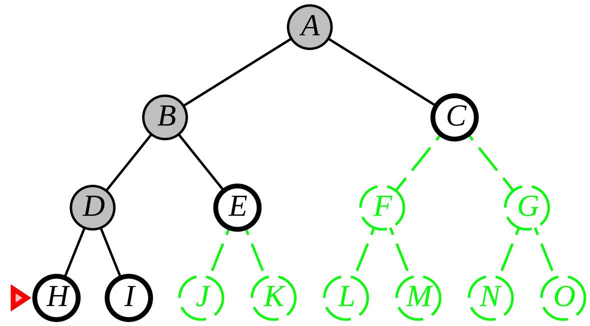
Depth-First Search

Expand deepest unexpanded node
Implementation: *fringe* = LIFO queue, i.e., put successors at front



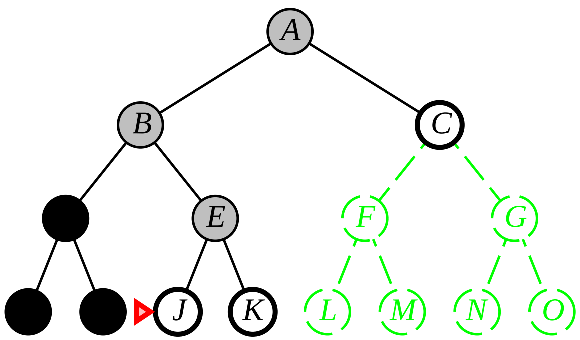
Depth-First Search

Expand deepest unexpanded node
Implementation: *fringe* = LIFO queue, i.e., put successors at front



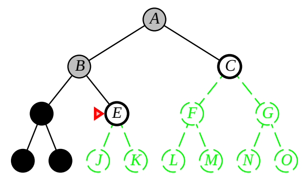
Depth-First Search

Expand deepest unexpanded node
Implementation: *fringe* = LIFO queue, i.e., put successors at front



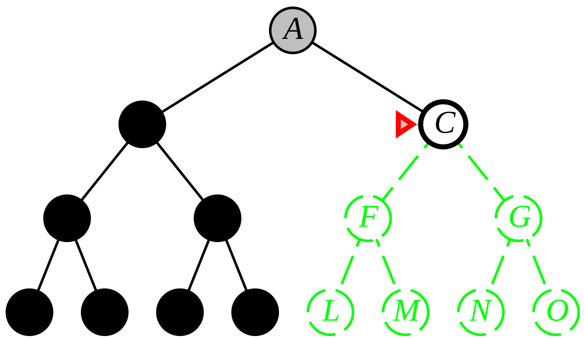
Depth-First Search

Expand deepest unexpanded node
Implementation: *fringe* = LIFO queue, i.e., put successors at front



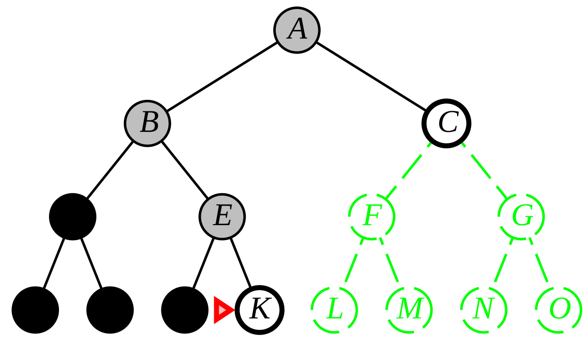
Depth-First Search

Expand deepest unexpanded node
Implementation: *fringe* = LIFO queue, i.e., put successors at front



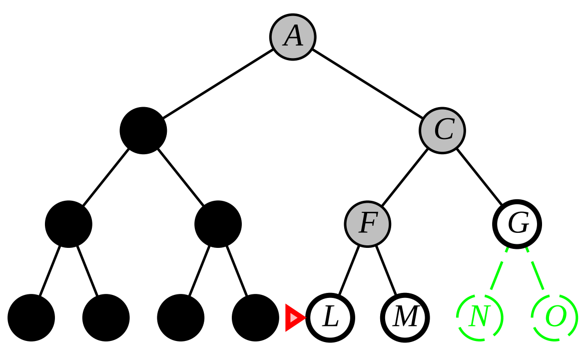
Depth-First Search

Expand deepest unexpanded node
Implementation: *fringe* = LIFO queue, i.e., put successors at front



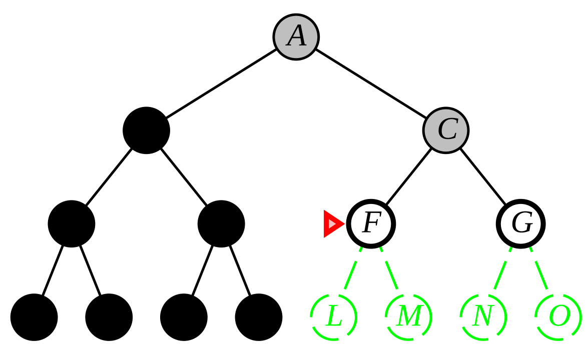
Depth-First Search

Expand deepest unexpanded node
Implementation: *fringe* = LIFO queue, i.e., put successors at front



Depth-First Search

Expand deepest unexpanded node
Implementation: *fringe* = LIFO queue, i.e., put successors at front

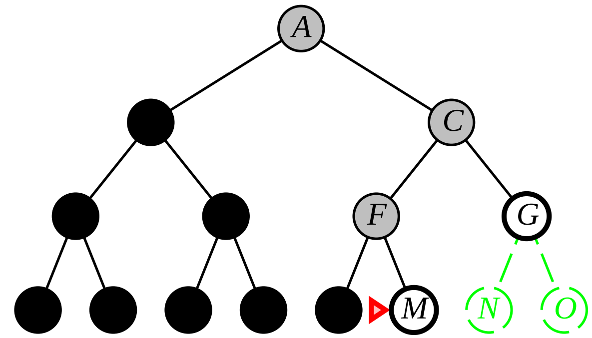


Properties of depth-first search

Complete??

Depth-First Search

Expand deepest unexpanded node
Implementation: *fringe* = LIFO queue, i.e., put successors at front



Properties of depth-first search

Complete?? No: fails in infinite-depth spaces, spaces with loops Modify to avoid repeated states along path $\Rightarrow$ complete in finite spaces
Time?? $O(b^m)$: terrible if m is much larger than d but if solutions are dense, may be much faster than breadth-first
Space??

Properties of depth-first search

Complete?? No: fails in infinite-depth spaces, spaces with loops Modify to avoid repeated states along path $\Rightarrow$ complete in finite spaces
Time??

Properties of depth-first search

Complete?? No: fails in infinite-depth spaces, spaces with loops Modify to avoid repeated states along path $\Rightarrow$ complete in finite spaces
Time?? $O(b^m)$: terrible if m is much larger than d but if solutions are dense, may be much faster than breadth-first
Space?? $O(bm)$, i.e., linear space!
Optimal?? No

Properties of depth-first search

Complete?? No: fails in infinite-depth spaces, spaces with loops Modify to avoid repeated states along path $\Rightarrow$ complete in finite spaces
Time?? $O(b^m)$: terrible if m is much larger than d but if solutions are dense, may be much faster than breadth-first
Space?? $O(bm)$, i.e., linear space!
Optimal??

Iterative Deepening Search

```
function Iterative-Deepening-Search (problem) return soln
    for depth from 0 to MAX-INT do
       result := Depth-Limited-Search(problem, depth)
       if (result != cutoff) then return result
    end for
end function
```


Depth-Limited Search

= depth-first search with depth limit l, i.e., nodes at depth l have no successors

```
function Depth-Limited-Search (problem, limit) return soln/fail/cutoff
   return Recursive-DLS(Make-Node(Initial-State(problem)), problem, limit)
end function

function Recursive-DLS (node, problem, limit) return soln/fail/cutoff
   cutoff-occurred := false;
   if (Goal-State(problem, State(node))) then return node;
   else if (Depth(node) == limit) then return cutoff;
   else for each successor in Expand(node, problem) do
           result := Recursive-DLS(successor, problem, limit)
           if (result == cutoff) then cutoff-occurred := true;
           else if (result != fail) then return result;
        end for
        if (cutoff-occurred) then return cutoff; else return fail;
end function
```

Iterative deepening search $l = 0$

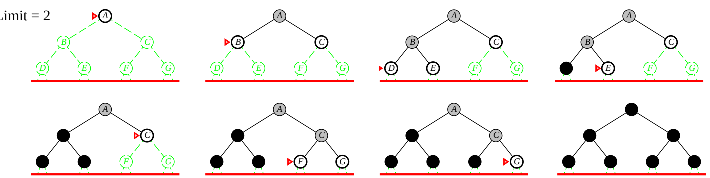
Limit = 0

Iterative deepening search $l = 1$

Limit = 1

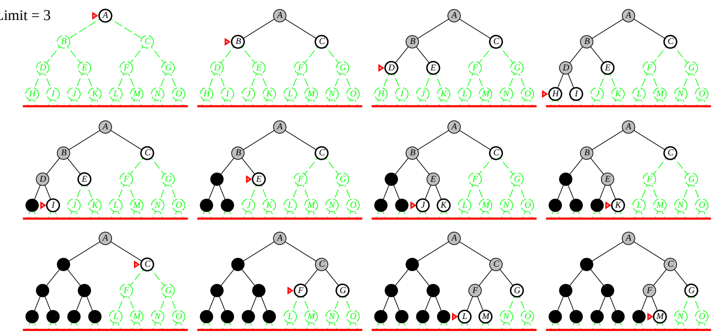
Iterative deepening search $l = 2$

Limit = 2



Iterative deepening search $l = 3$

Limit = 3



Properties of iterative deepening search

Complete??

Properties of iterative deepening search

Complete?? Yes
Time??

Properties of iterative deepening search

Complete?? Yes
Time?? $(d+1)b^0 + db^1 + (d-1)b^2 + \ldots + b^d = O(b^d)$
Space??

Properties of iterative deepening search

Complete?? Yes
Time?? $(d+1)b^0 + db^1 + (d-1)b^2 + \ldots + b^d = O(b^d)$
Space?? $O(bd)$
Optimal??

Summary of Algorithms

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening
Complete?	Yes*	Yes*	No	Yes, if $l \geq d$	Yes
Time	b^{d+1}	$b^{\lceil C^*/\epsilon \rceil}$	b^m	b^l	b^d
Space	b^{d+1}	$b^{\lceil C^*/\epsilon \rceil}$	bm	bl	bd
Optimal?	Yes*	Yes*	No	No	Yes

Properties of iterative deepening search

Complete?? Yes
Time?? $(d+1)b^0 + db^1 + (d-1)b^2 + \ldots + b^d = O(b^d)$
Space?? $O(bd)$
Optimal?? Yes, if step cost = 1 Can be modified to explore uniform-cost tree

Numerical comparison for $b = 10$ and $d = 5$, solution at far right:

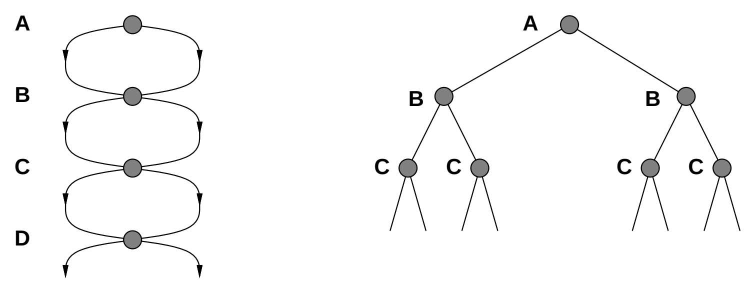
$$N(\text{IDS}) = 50 + 400 + 3,000 + 20,000 + 100,000 = 123,450$$
$$N(\text{BFS}) = 10 + 100 + 1,000 + 10,000 + 100,000 + 999,990 = 1,111,100$$

Summary

- Problem formulation usually requires abstracting away real-world details to define a state space that can feasibly be explored
- Variety of uninformed search strategies
- Iterative deepening search uses only linear space and not much more time than other uninformed algorithms

Repeated states

Failure to detect repeated states can turn a linear problem into an exponential one!



Example: Romania

For a given strategy, what is the order of nodes to be generated (or stored), and expanded? With or without checking duplicated nodes?

- Breadth-first
- Depth-first
- Uniform-cost
- Depth-limited
- Iterative-deepening