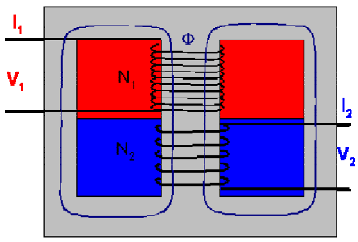
7. Transformers

- 7.1 What does one look like?
- 7.2 How does it work?
- 7.3 What is the model?
- 7.4 How does one find the parameters?
- 7.5 How are transformers used?
- 7.6 What's missing?

7.1 What does one look like?

- Primary winding
  - V_1, I_1, N_1.
- Secondary winding
  - V_2, I_2, N_2.
- HV & LV
- Core
  - Flux Φ links 1 and 2



7.2 How does it work?

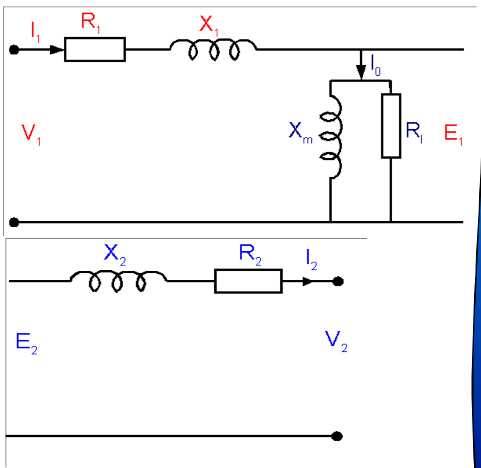
- Simplified Approach- *IDEAL* Transformer:
- Flux, EMF:
  - AC Voltage, V_1 applied to primary, secondary OC.
  - Small AC current, I_0, flows $\Rightarrow \mathcal{F}_0 \Rightarrow \Phi$.
  - $E_1 = N_1 d\Phi/dt = 4.44\, f\, N_1\Phi \approx V1$.
  - $E_2 = N_2\, d\Phi/dt = 4.44\, f\, N_2\Phi \approx V_2$.
  - $\mathbf{V_1/V_2 = N_1 / N_2 = k}$

Transformer action

- Balance of Ampere Turns:
- Load R_2.
  - I_2 flows $\Rightarrow$ $\mathcal{F}_2$ $\Rightarrow$ decrease in $\Phi \Rightarrow$ $\downarrow E_1$.
  - BUT $E_1 \approx V_1$ so $\uparrow I_1$, such that $\mathcal{F}_1 = \mathcal{F}_2$.
  - $N_1 I_1 = N_2 I_2$
  - $\mathbf{I_1 / I_2 = N_2 / N_1 = 1/k}$
- Effective load on primary: R'_2:
  - $R'_2 = V_1/I_2 = kV_2/(I_2/k) = k^2\, V_2/I_2$.
  - $R'_2 = k^2 R_2$

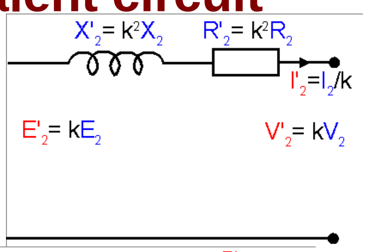
7.3 What is the model?

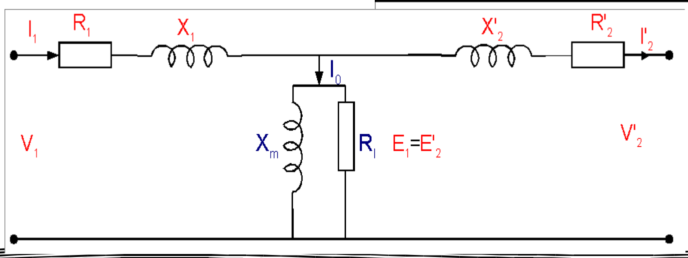
- Primary
  - R_1
  - X_1
  - X_m or B
  - R_I or G
- Secondary
  - X_2
  - R_2



Full equivalent circuit

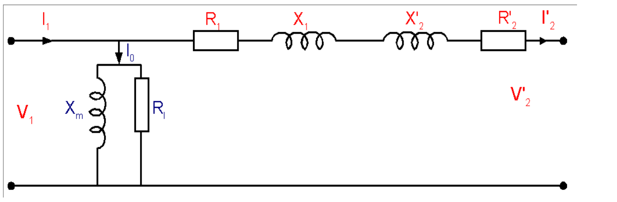
- Refer primary to secondary (can also refer to secondary)
- $E_1 = E'_2$.

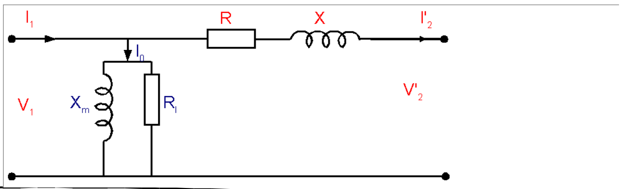




Approximate Equivalent Circuit

- *If* $R_1, X_1 \ll X_M, R_I$ then move parallel branch to left.





7.4 How does one find the parameters?

- Open circuit test (LV)

$$R_I = \frac{V^2}{P}$$

$$I_x = \sqrt{I^2 - \left(\frac{V}{R_I}\right)^2}$$

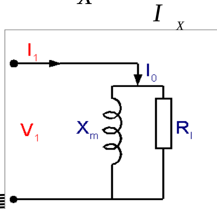
$$X = \frac{V}{I_x}$$

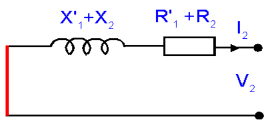
Short Circuit test (HV)

$$R = \frac{P}{I^2}$$

$$Z = \frac{V}{I}$$

$$X = \sqrt{Z^2 - R^2}$$





7.5 How are transformers used?

- Ratings
- Performance parameters
- Three Phase connections
- Special Transformers

Ratings

- *Rating* is NOT necessarily *operating condition*.
- Main (Nameplate)
  - Voltage
  - VA
- Secondary
  - Frequency
  - % Impedance = $\frac{I_{Rated}}{I_{SC}} * 100$
    - $Z = V_p/I_{sc}$

Performance parameters

- Efficiency

$$\eta = \frac{P_{Out}}{P_{In}} = \frac{P_{Out}}{(P_{Out} + P_{Cu} + P_{Fe})}$$

$$\eta = \frac{P_{out}}{(P_{out} + I^2 R + P_{Fe})}$$

$$\eta_{max} \Rightarrow P_{Cu} = P_{Fe}, unity\ pf$$

- % Regulation: $\frac{V_{NL} - V_L}{V_{NL}} * 100$

Three Phase connections

- 3 single phase transformers or 1 three phase
- Connections:
  - Star-star
  - Delta - delta
  - Star-delta, delta-star
- Always work in *PHASE* quantities. Remember √3

Special Transformers

- Power transformer
- Distribution
- Welding

Example

- A 500 kVA, 3 Phase 22kV, 11kV star-star transformer has a no load current of 1.1A. It has a maximum efficiency of 96% when operating at rated load, unity power factor. The short circuit current is 8 times rated current.
- Determine:
  - the parameters of the approximate equivalent circuit.
  - the regulation when supplying 80% rated current at 0.8 pf lag

Solution

- $V^*_p = 22k/\sqrt{3} = 12.7$ kV $I^*_p = S/3V_p = 500/12.7 = 39.4A$
- Approximate Circuit
  - $P_{Out} + 2P_{fe} = P_{Out}/\eta \Rightarrow P_{fe} = 0.5(1/\eta - 1)P_{Out} = 10$ kW
  - $R_I = 3V_p^2 / P_{fe} = 3\ 12710^2 / (10000) = 48.5$ kΩ
  - $I_R = V_p / R_I = 0.26A$
  - $X_M = V_p / \sqrt{(I_0^2 - I_R^2)} = 12700/\sqrt{(1.1^2 - 0.26^2)} = 11.9$ kΩ .
  - $R = P_{cu}/3I_p^2 = 10000/(3\ 39.4^2) = 2.1\Omega$.
  - $Z = V_p/(8\ I_p) = 12700/(8\ 39.4) = 40.3\ \Omega$
  - $X = \sqrt{(Z^2 - R^2)} = 40.2\Omega$

Solution part 2

- Solve for V'_2: $V_1 = 12.7$ kV $I = 0.8 \cdot 39.4 = 31.5$A
- $V_1 \angle\alpha = V'_2 \angle 0 + I\angle{-36.8}\,(R+jX)$
- Imaginary: $V_1 \sin\alpha = IR \sin -36.8 + IX \cos -36.8$
- $\alpha = \sin^{-1}((-39.6+1014)/12700) = 4.8°$
- Real: $V_2 \cos\alpha = V'_2 + IR \cos -36.8 - IX \sin -36.8$
- $V'_2 = V_2 \cos\alpha - IR \cos -36.8 + IX \sin -36.8 = 11.9$ kV
- Regulation: $(12.7-11.9)/12.7 = 6.3\%$

7.6 What's missing?

- Alternative connections
- Saturation
- Inrush currents
- Interwinding capacitance
- Zig-zag windings etc.