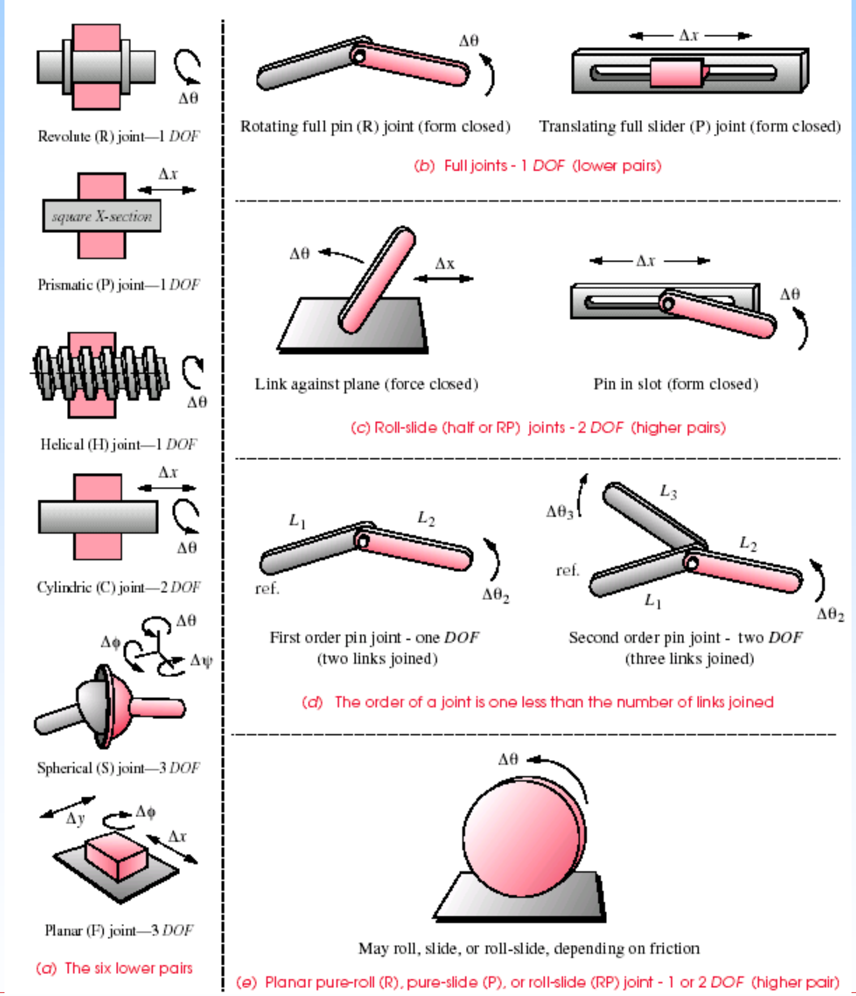
Mobility/ Degree of Freedom

Lecture 2

From Last Lecture: Kinematic pairs

Joints restraint and allow motion between links

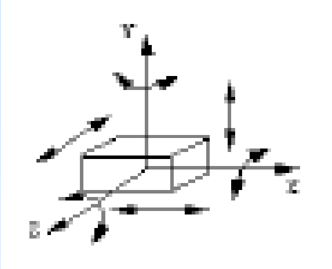


Today's Agenda

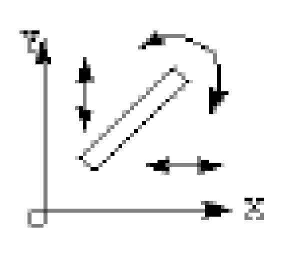
- Definition Degrees of Freedom
- Determining Degrees of Freedom/mobility of a mechanism
- Mechanisms and Structures
- Mobility Analysis of Mechanisms

Simple Examples

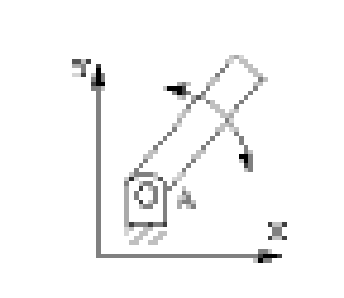
A link free in space has six DOF since it is free to translate in each of the X, Y, and Z directions and to rotate about each of the axes.



In planar linkages, motion is restricted to the XY plane. The DOF is hence reduced from six to three



If we attach this link to the ground with a single DOF pair, such as a turning pair, then its DOF is further reduced from three to one



Mobilty of Linkages

Definition:

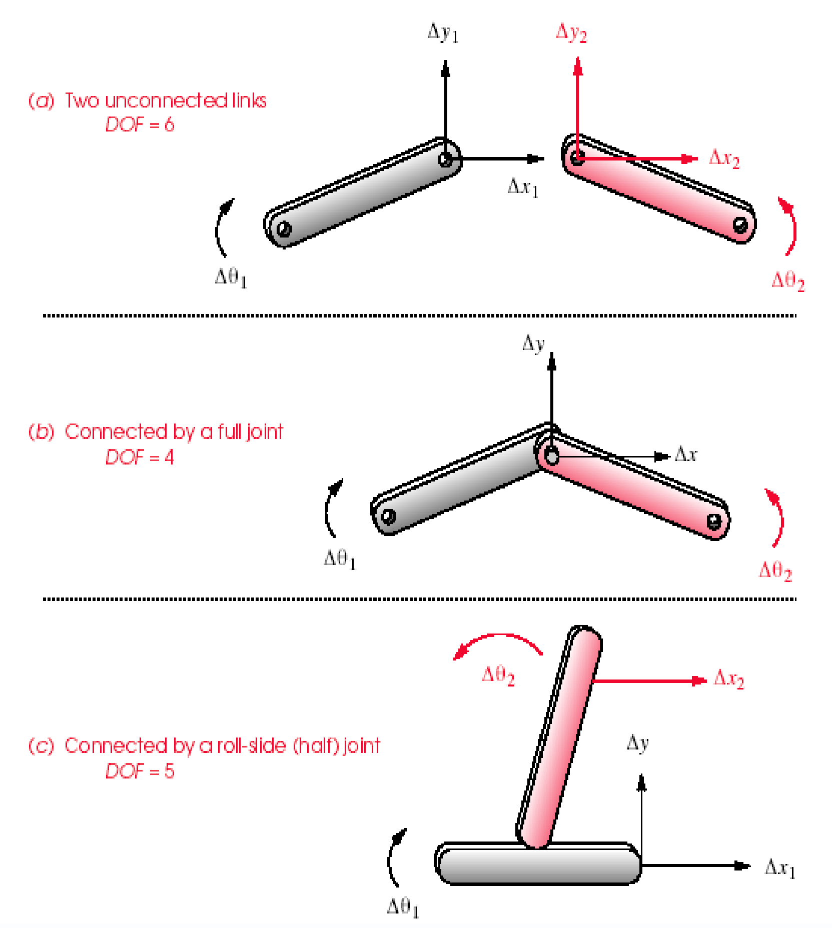
- The mobility of a mechanism can be defined as the minimum number of independent parameters required to specify the location of every link within a mechanism.
- Mobility equals the total number of degrees of freedom (DOF)

DOF or Mobility of Kinematic pairs

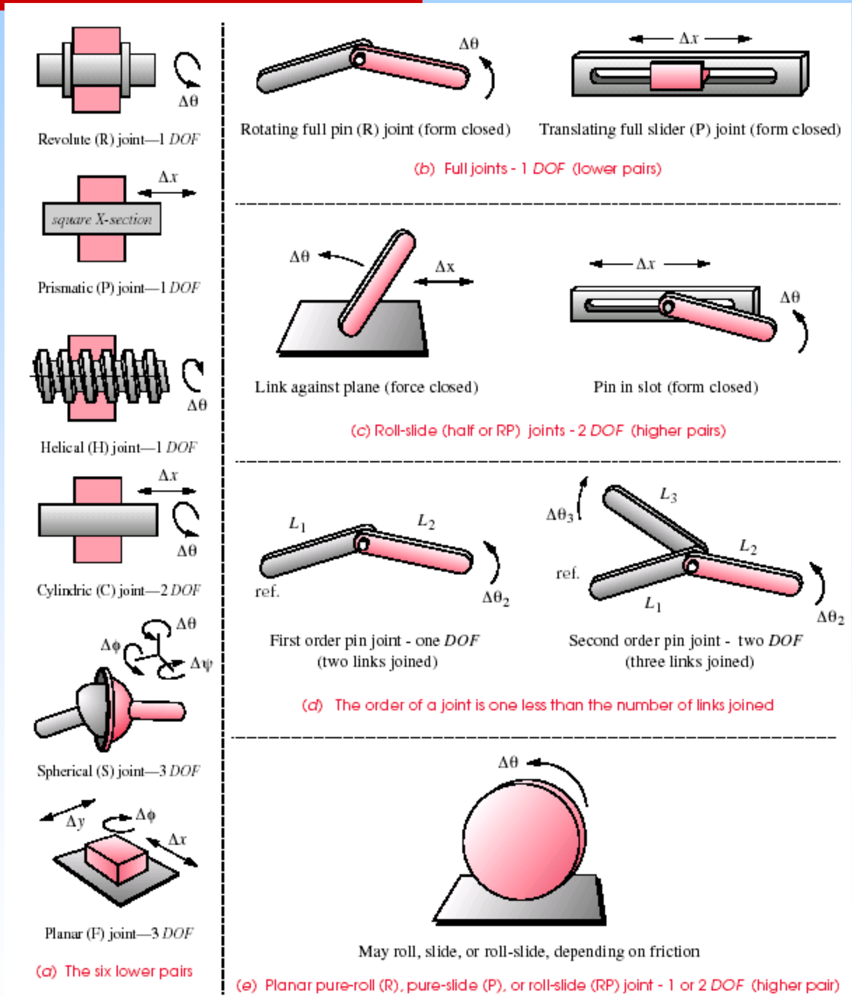
In planar linkages, motion is restricted to the XY plane. The mobility is hence reduced from six to three for each link.

Attaching these 2 links together with a *full* single DOF pair, such as a turning or sliding pair, then its mobility is further reduced from three to two for each link.

Attaching these links with a half rolling and sliding 2_DOF pair, such as a rolling and sliding pair, then the total number of mobility is reduced by one from six to five.

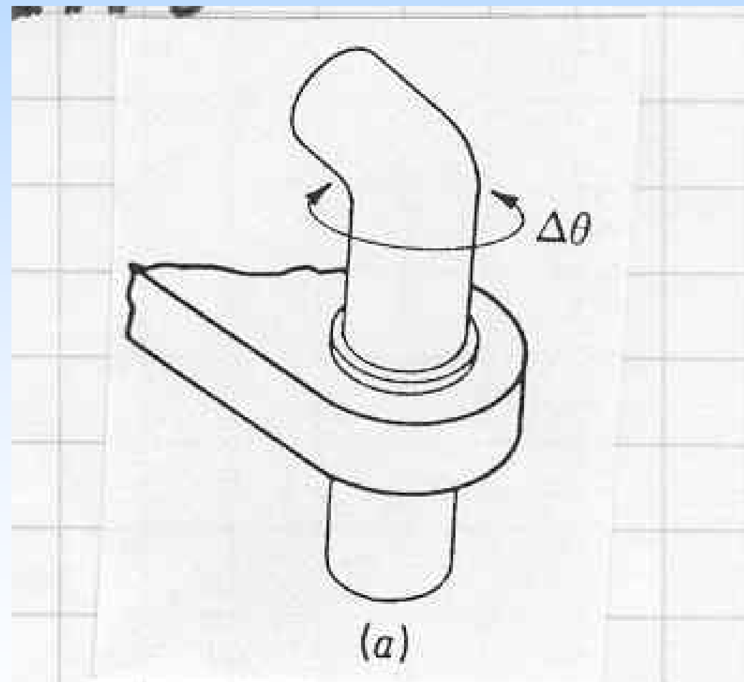


DOF or Mobility of Kinematic pairs



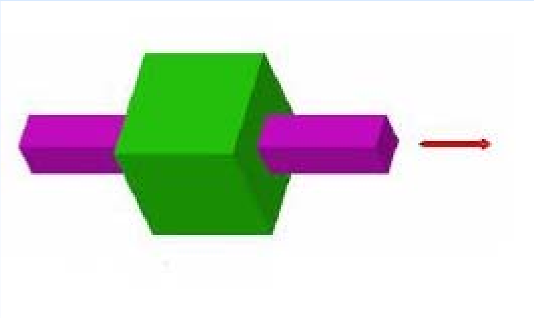
Turning Pair or Revolute

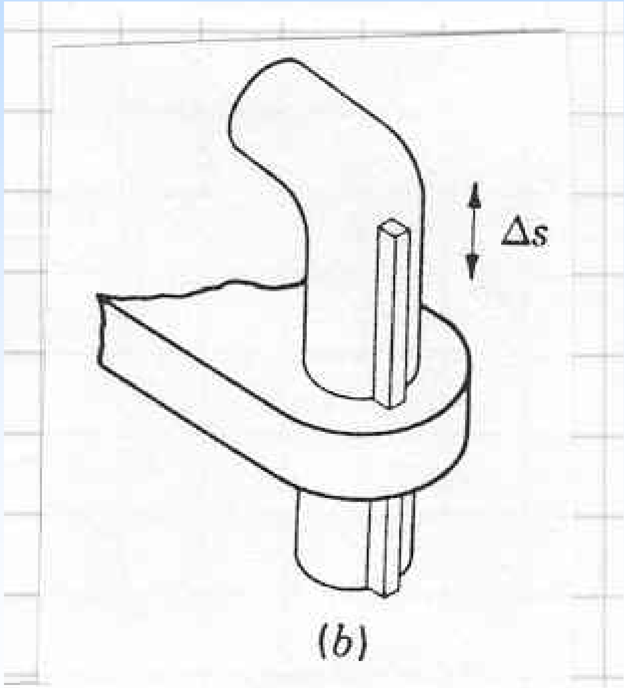
- Permits only relative rotation
- 1 DoF → Full Joint
- Lower Pair
- (R)



Prismatic Pair

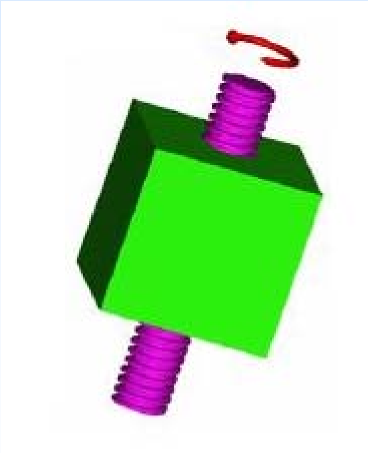
- Permits only relative sliding motion and therefore is often called a sliding joint
- Has 1 DoF → Full Joint
- Lower Pair
- (P)

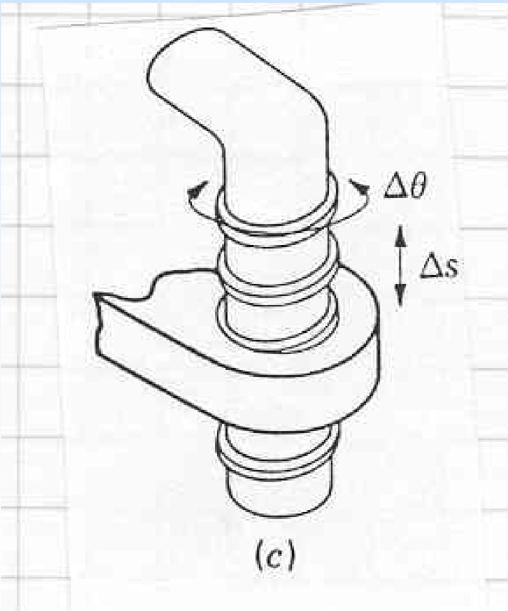




Screw Pair or Helix Pair

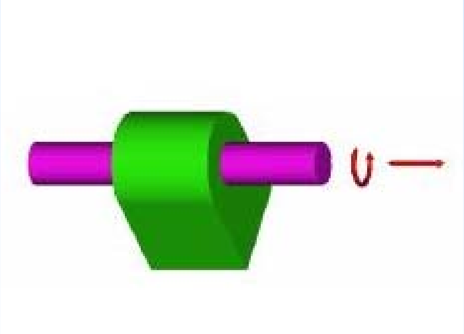
- Has only 1 DoF because the sliding and rotational motions are related by the helix angle and thread
- 1 DoF → Full Joint
- Lower pair
- (RP)→(H)

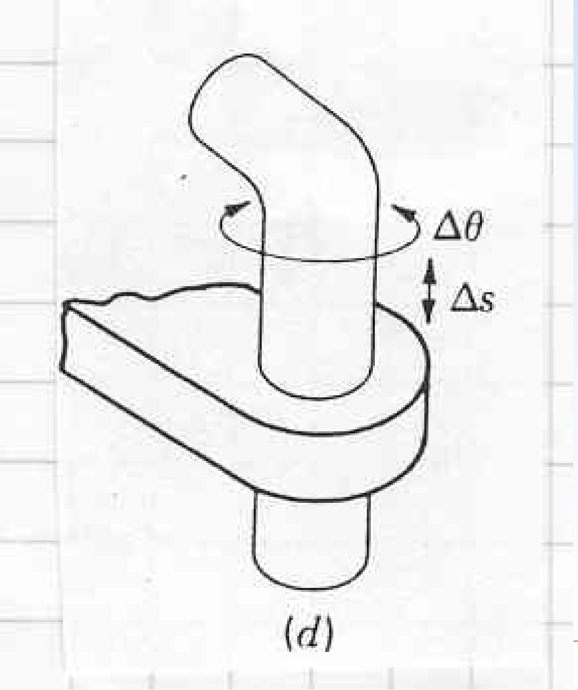




Cylindrical Pair

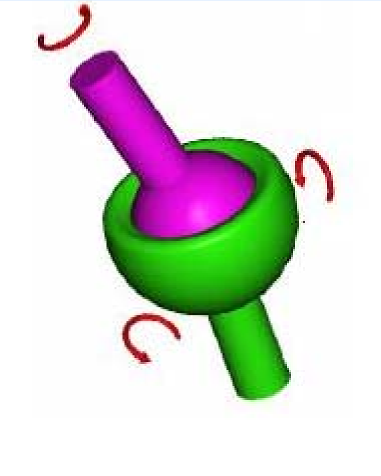
- Permits both angular rotation and an independent sliding motion.
- 2 DoF (1 translation & 1 Rotation) → Half Joint
- Lower Pair
- (RP)→(C)

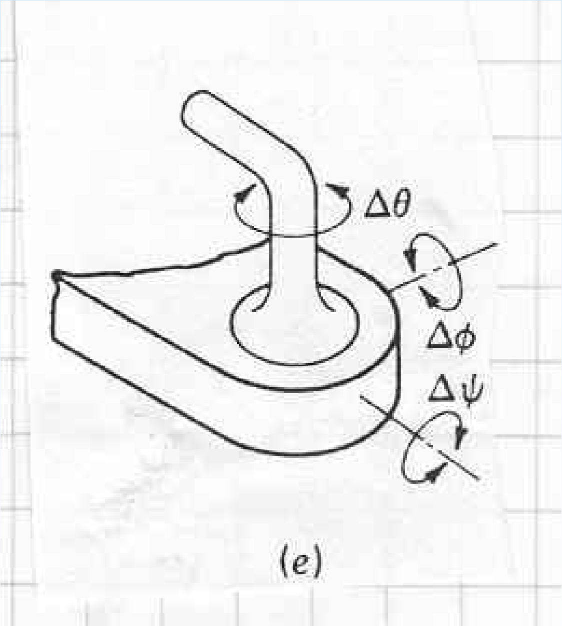




Globular or Spherical Pair

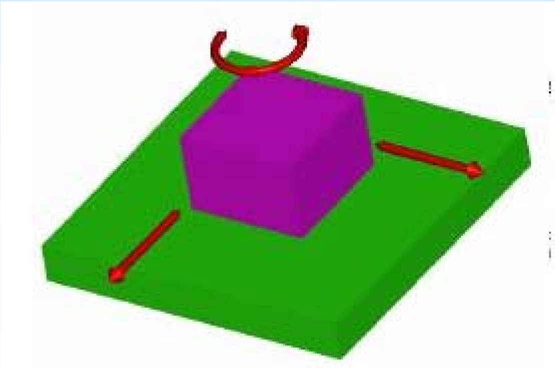
- Ball-and-socket joint thaqt has rotation about each of the coordinate axes
- 3 DoF
- Lower Pair
- (RRR)→(S)

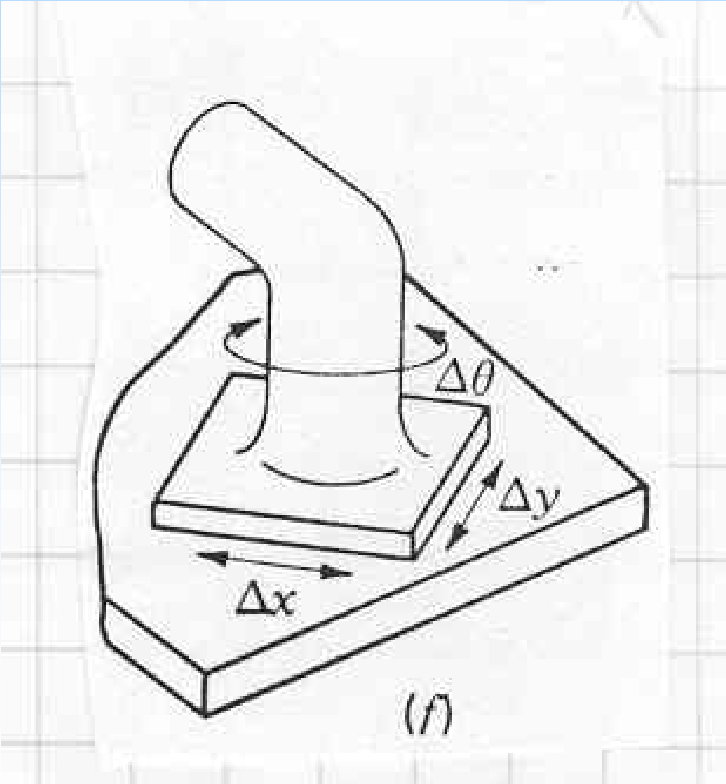




Flat Pair or Planar Pair

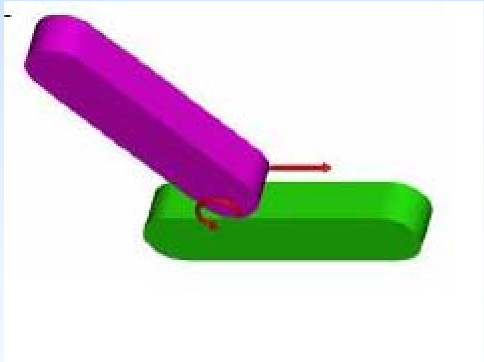
- Seldom found in a mechanism
- 3 DoF
- Lower Pair
- (RPP)→(F)

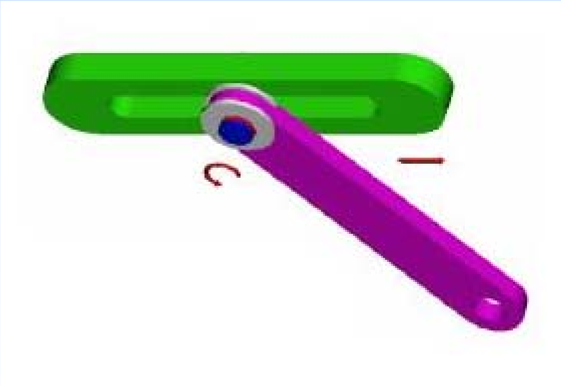




2 DOF Joints

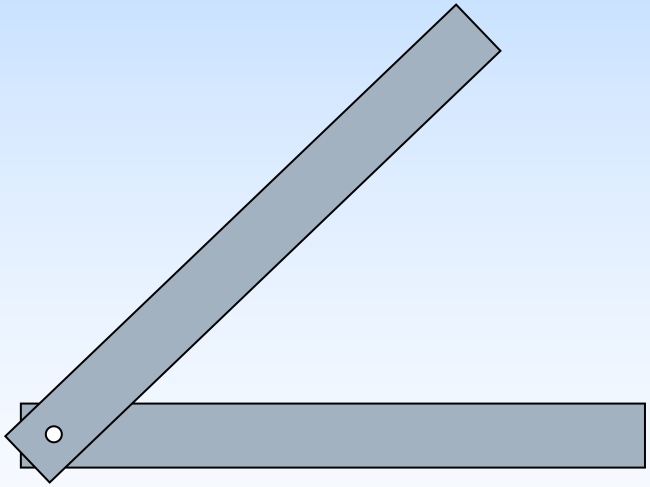
- Half Joint: in planar problems removes 1 DOF
- higher pair: joints with point or line contact



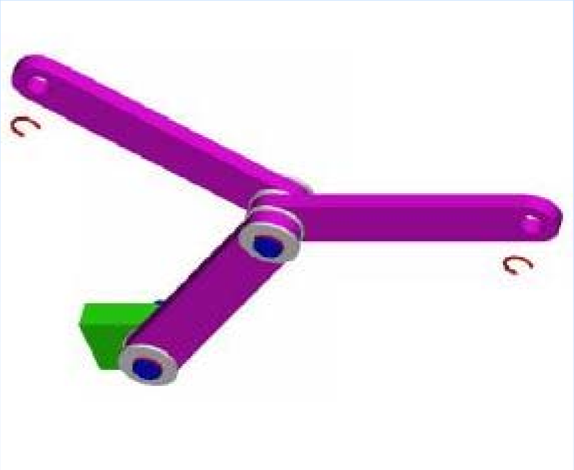


Number of Links at a Joint

- The joint order is defined as the number of links joined minus one



1 DOF



2 DOF

Determining Mobility or DOF

Gruebler & Kutzbach Equations

In a mechanism rigid bodies may be constrained by different kinds of planar pairs. If

- n = number of links (including the frame)
- j_1 = number of lower pairs (allowing one DOF)
- j_2 = number of higher pairs (allowing two DOF),

then

- m = mobility of the mechanism can be determined using the *Gruebler & Kutzbach* criterion:

$$m = 3(n-1) - 2J_1 - J_2$$

Practical Interpretations of Mobility

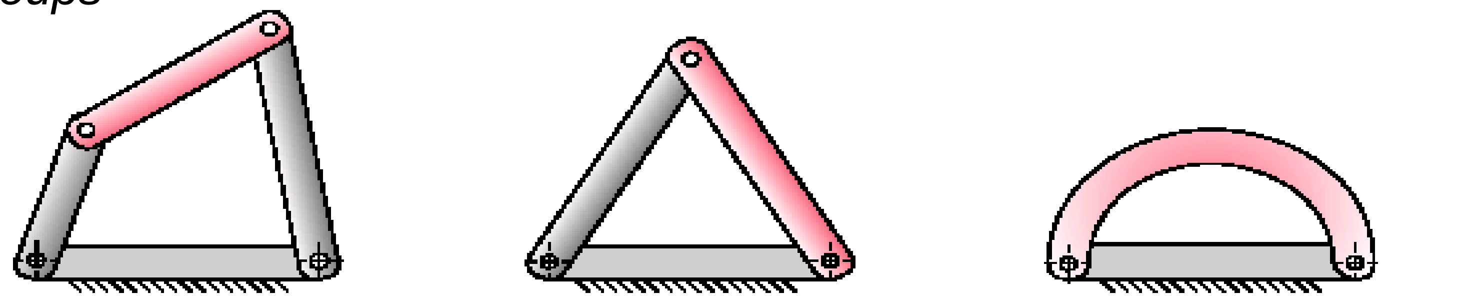
Steps involved in applying calculating mobility.

- Count number of elements/links: n
- Count number of single dof pairs: j_1
- Count number of two dof pairs: J_2
- $m = 3(n - 1) - 2j_1 - J_2$

- Classify into mechanism, structure or statically indeterminate system
- Any joint at which k links meet must count as k–1 joints.
- Cases of slip and no-slip must be considered for rolling pairs.
- Grubler's criterion can be easily fooled because it does not take geometry into account.

Practical Interpretations of Mobility

The mobility of a device can be used to classify it into the following groups



(a) Mechanism—DOF = + 1 (b) Structure—DOF = 0 (c) Preloaded structure—DOF = −1

***m* = 0:** motion impossible – system has enough constraints at the joints necessary to ensure equilibrium.

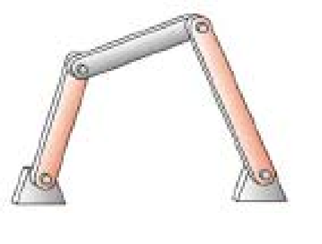
***m* ≥ 1:** Mechanism

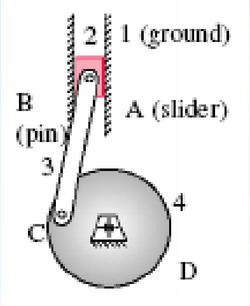
***m* = -1:** Statically indeterminate structure. Links have more constraints than are needed to maintain equilibrium (TRUSS).

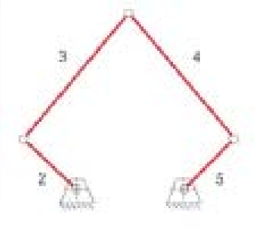
Determining Mobility or DOF

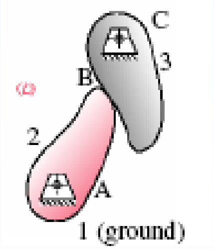
Grubler & Kutzbach Equations

1. Count links
2. Count joint
3. Apply equation



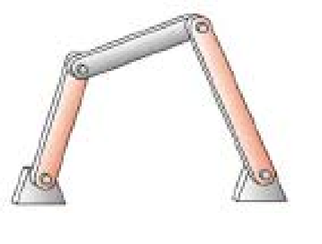


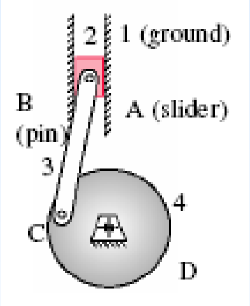


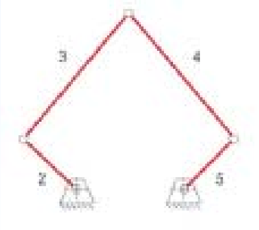


Determining Mobility or DOF

Grubler & Kutzbach Equations

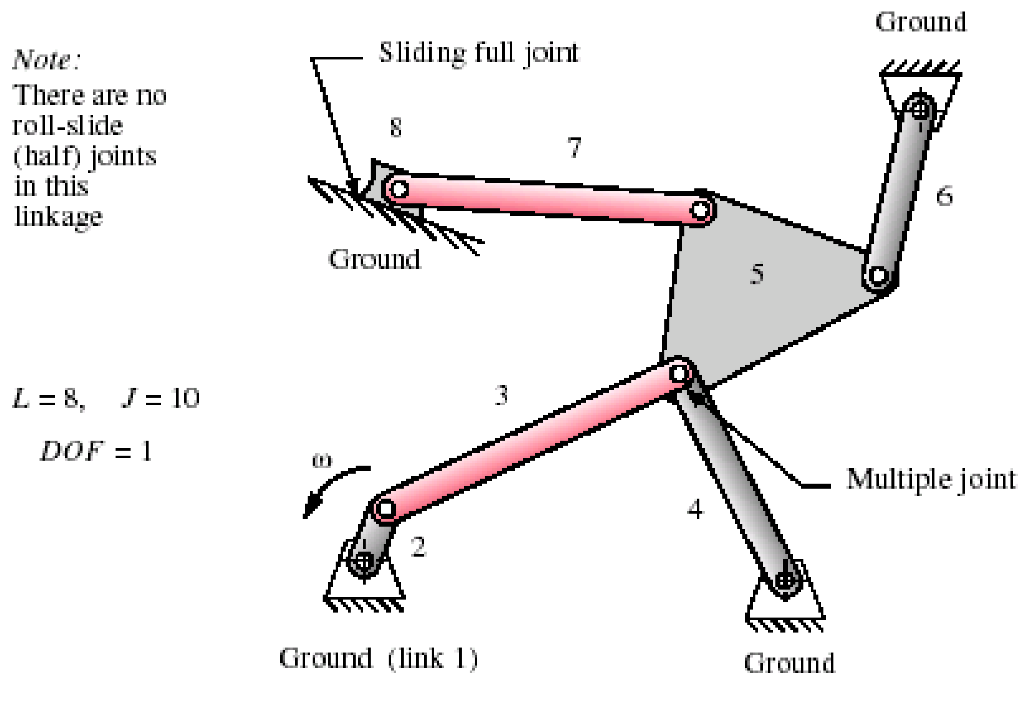






1. Count links
2. Count joint
3. Apply equation

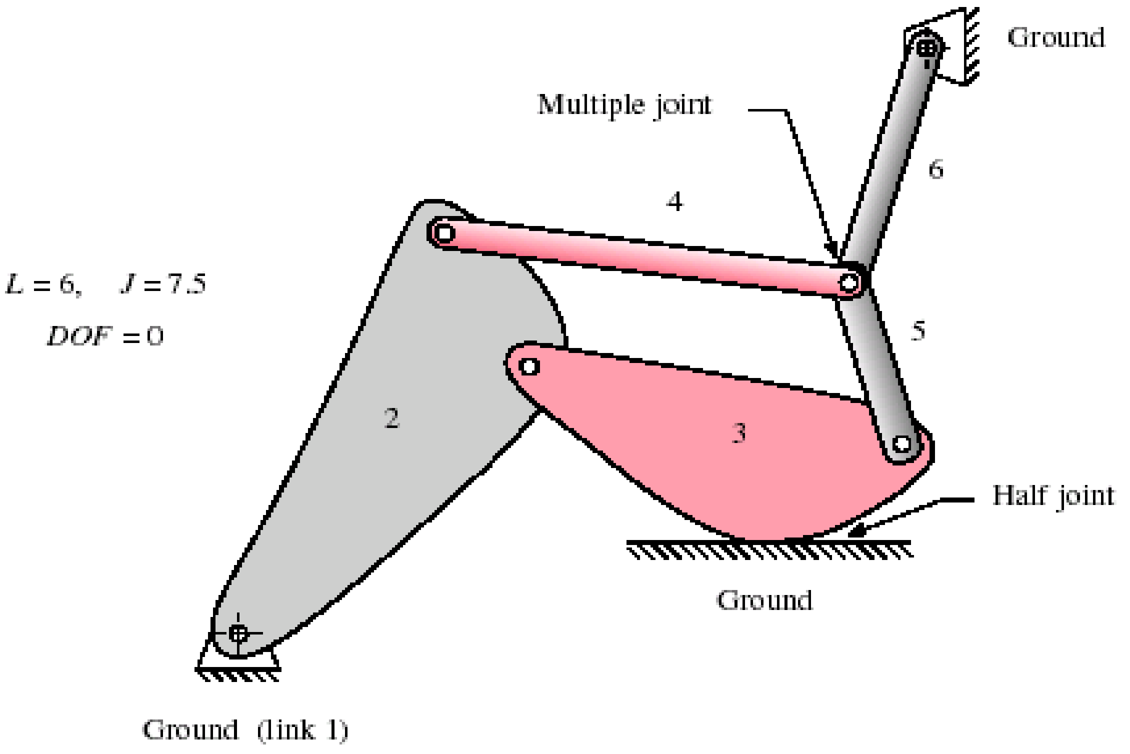
Applying Mobility Equations - 1



(*a*) Linkage with full and multiple joints

$$M = 3(L-1) - 2(J1) - J2 = 3(8-1) - 2(10) - 0 = 1$$

Applying Mobility Equations - 2



(b) Linkage with full, half, and multiple joints

$$M = 3\ (L-1) - 2(J1) - J2 = 3(6-1) - 2(7) - 1 = 0$$

Practical Interpretations of Mobility

- Although the Grubler's formula is is useful in determining the mobility of a wide variety of commonly used engineering mechanisms. It still:
  - yields only a ***theoretical*** result.
  - does not take into account the *geometry* of the links or mechanism.
  - can be misleading and,

Therefore, when ambiguous result, the actual mobility of a mechanism must be determined by inspection.

Practical Interpretations of Mobility

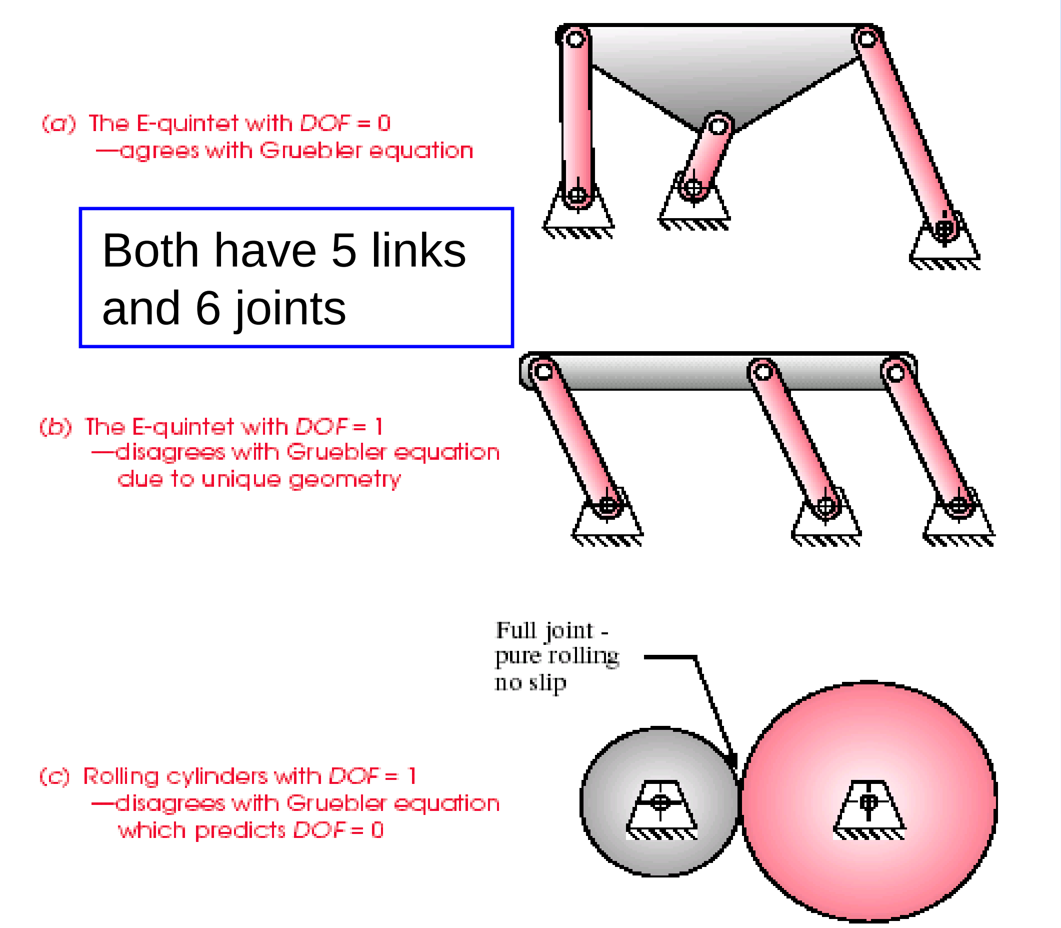
The mobility of a linkage can also be defined as the number of independent inputs needed in order to produce a controlled output.

– In order to control a mechanism, the number of independent input motions (i.e. motors) must equal the number of DOF of the mechanism.

– For a four-bar or slider-crank mechanism, only one input motor is required for control because ***m* = 1**.

Gruebler Paradoxes

Gruebler's equation does not account for link geometry, in rare instance it can lead to misleading result



Summary

- Gruebler's formula
  - Single-dof joints
  - Two-dof joints
- Homework:
  - Problems 1.1a, 1.2a &1.2b, 1.5a
- Next Monday
  - Inversion & Grashof's Criterion